One of the main aims of the present project is to find methods of designing teaching material that stimulates the development of thinking abilities more effectively. In the closing phase of this work, an experiment was carried out in two age groups in regular Hungarian schools to examine the effectiveness of the modified teaching material devised previously. The main principles of this work were that only the contents of the regular teaching material could be used to form structured tasks and exercises and that all the developmental effects should be integrated into the teaching material itself. The developmental activities should not be applied outside the regular framework of the curriculum and extra time should be devoted to the developmental work.

In traditional education, chiefly mathematics and some formal grammar exercises were regarded as the best ways to form the thinking. The early Piagetian framework supported these tendencies by postulating general stages and structures and universal developmental patterns. The most remarkable shift in thinking about cognitive development is the growing importance of specific structures, content domains, and contexts. Two of the recent theories are especially close to our view: Fischer’s skill theory (Fischer 1980; Bidell and Fischer, Chapter 1 of this volume) and experiential structuralism (Demetriou and Efklides 1988; Demetriou, Gustafsson, Efklides, and Platsidou, Chapter 5 of this volume). While we cannot expect that the structures acquired in a certain content domain are generalized to every other content, we aim to point out the most general operational structures that are relevant to the most important content domains of school learning. On the other hand, we presume that any school subject can provide several developmental effects if its content is reorganized and enriched with special structured tasks. As a consequence of this view, we have devised different training materials for specific parts of operational thinking and organized separate training groups to test these materials.

Many studies have dealt with the development of operational thinking in the framework of the Piagetian tradition, and many examine the structures that are at the centre of our experiment.
Studies that involve training in the binary operations of propositional logic or combinatorial reasoning, or operational thinking in the context of school subjects, are closer to the scope of this chapter. Fishbein, Pampu, and Minzat (1970) showed that even 10-year-old students were able to produce the appropriate permutations and arrangements after they had been taught to use tree diagrams. Siegler, Liebert, and Liebert (1973) trained 10-year-olds to solve the Piagetian pendulum problem. Case (1974) proved that 8-year-olds can be taught to use the control of variables strategy. Collis (1980) describes the role of mathematics teaching in the development of operational thinking, while Jurd (1978) analyses history-type material from the same aspects. These studies lead to the question of the utilization of the teaching material to improve students' operational thinking.

Our experiment shows many similarities with the CASE project (Cognitive Acceleration through Science Education: Shayer 1987; Shayer and Adey 1988). We consider the most important characteristics of the CASE project to be (1) it uses a much longer intervention period than is common, (2) the experiments take place in regular school classes, and (3) they are based on school-related tasks. Over a period of two years, up to thirty intervention lessons were presented to students aged 11 and 12 at the beginning of the intervention. The results of these studies indicated that boys gained more than girls from the interventions, and the method worked better in classes where the lessons were given by experienced researchers.

In the present experiment the original Piagetian system of formal operations was re-formulated and three groups of operations were identified: the group of logical operations contains the binary operations of propositional logic; the system of combinatorative operations is enhanced by taking into account further combinatorial structures not studied by the Geneva school; and the group of systematizing operations contains the operations of ordering (binary relations), class inclusion, classification, and multiple classification. On the basis of these systems, paper and pencil test-batteries were devised and comprehensive assessments were carried out into the structure and development of these operational abilities. The results of this phase of the research have been communicated at several conferences (Csapó 1985) and have been published in three volumes in Hungarian (Nagy 1988; Csapó 1988; Csirikné 1989).

On the basis of this earlier work the teaching material of some school subjects was analysed and methods were devised for the improvement of the children's operational abilities. The present one-year experiment was designed to study the changes caused in the student's cognition by these methods. Of course, it is to be stressed that experiments carried out in the regular school context face many problems which are not important for laboratory experiments. These problems may lead to negative findings and it is important to be aware of them. Specifically, excluding the trivial cases
of experimental errors, some of the main possible causes of negative findings may be classified as follows:

1. The particular operational scheme was not modifiable at all.
2. The operational scheme has been modified, but the change could not be detected either (a) because the instrument was not sensitive enough to capture this kind of change or (b) because the effect was delayed, and could be detected only a long time after the intervention.
3. The operational scheme was not modified because the treatment applied was ineffective, ill-conceived, or misapplied.

Taking into account the methodological problems mentioned above, our strategy in this work was to organize the experiment in an ecologically valid environment in as standard a way as the practical limits allow, but at the same time to control these limits and problems in as many ways as possible. We have to adapt the design to these limits, and we must not forget them during the interpretation of the results either.

**METHOD**

**Design**

In order to reduce the ambiguity in the interpretation of the findings, we applied a complex experimental design where ages and interventions were systematically varied though the same overall concept of treatment was used. The three basic dimensions of the experimental design were the operational abilities to be improved, the age of the students, and the developmental influences (modified teaching material) on the students. We chose grammar and science in the fourth grade (10-year-olds) and chemistry and physics in the seventh grade (13-year-olds) for the purposes of experimental instruction (Table 8.1). For every combination of the four groups of subjects and the three abilities, a particular developmental task-system was devised. This gives twelve different ways of experimental instruction. In some experimental groups, the interventions were applied in both school subjects (science and grammar or chemistry and physics). This enables one to study the interaction between the different kinds of interventions. At both ages, a control group was also involved. There were three classes in the experimental groups and six classes in the control groups.

**Subjects**

Urban, suburban, and rural areas were represented in the twenty-eight schools taking part in the experiment. The experimental classes within
these schools were selected in co-operation with the school principals and class teachers. Specifically, the principals were asked to suggest ‘average’ classes for both the experimental and the control groups. We opted to exclude classes regarded as ‘excellent’ or ‘very poor’, because, according to our pilot work, the effect of intervention may be influenced significantly by the starting level of the students. The control classes were selected from the same schools, preferably seventh-grade controls from the schools where the experiment took place in the fourth grade, and vice versa. As the results showed later, despite our intentions, in some of the schools principals and teachers preferred to offer the ‘better’ classes for the experimental instruction.

At the time of the first measurement (September 1987), the mean age for all the fourth graders (experimental+control, n=930) was 9.68 years (SD=.49). The mean age of the seventh graders (n=890) was 12.59 years (SD=.44).

No classes dropped out during the experimental year: all fifty-four intervention classes were considered, as they completed the experimental instruction. Although examination of the documentation made by the teachers in some cases led to the suspicion that they had not done exactly what they had been expected to do, these classes were not dropped either. In three experimental classes the teachers changed during the course of the

Table 8.1 The system of the experimental groups

<table>
<thead>
<tr>
<th>School subjects involving experimental instruction</th>
<th>Abilities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Combinative</td>
</tr>
<tr>
<td>4th grade</td>
<td></td>
</tr>
<tr>
<td>Grammar</td>
<td>XXX</td>
</tr>
<tr>
<td>Science</td>
<td>XXX</td>
</tr>
<tr>
<td>Grammar and science</td>
<td>XXX</td>
</tr>
<tr>
<td>4th grade control</td>
<td>XXXXX</td>
</tr>
<tr>
<td>7th grade</td>
<td></td>
</tr>
<tr>
<td>Chemistry</td>
<td>XXX</td>
</tr>
<tr>
<td>Physics</td>
<td>XXX</td>
</tr>
<tr>
<td>Chemistry and physics</td>
<td>XXX</td>
</tr>
<tr>
<td>7th grade control</td>
<td>XXXXX</td>
</tr>
</tbody>
</table>

X=1 class
year and the new teachers had to go on with the intervention procedure. This may have disturbed the experimental work, too. These effects were also considered as an integral part of the usability of the experimental methods.

A data sheet and nine tests were administered before the beginning of the interventions and five tests after the interventions. In some cases, several unexpected events made it impossible to organize a proper testing session. These cases are handled as missing data. Specifically, out of a total of 990 testings (66 classes x 10 pre- and 5 post-testings), nineteen were lost.

**Measurement instruments**

A variety of evidence was collected before and after the interventions. Only the characteristics of the tests used to measure the developmental levels of the abilities dealt with will be presented here. The system of variables can be summarized as follows:

1. **Dependent (experimental) variables**
   1.1 Logical Operations Test
   1.2 Combinative Operations Test
   1.3 Systematizing Operations Test

2. **Mental background**
   2.1 Cognitive domain
      2.1.1 Intelligence: Raven’s Matrices
      2.1.2 School achievements: school marks in the main school subjects
   2.2 Affective domain
      2.2.1 Motivation (Kozéki and Entwistle 1984)
      2.2.2 Self-concept: effectiveness and talent (Jerusalem 1984)
      2.2.3 Test anxiety (‘TAI H/C’, Sipos, Sipos, and Spielberger 1985)

3. **Social background/environment**
   3.1 Family
      3.1.1 Social economic status
      3.1.2 Family structure
   3.2 Characteristics of living area

For the measurement of developmental levels of operational abilities, special test batteries were devised based on our earlier research. In this way, the present findings can be compared with those obtained before.

The Systematizing Operations Test was developed and validated by Nagy (1988). A short version of this test was used in the present research. It consists of nineteen dichotomously scored tasks (e.g., seriations, class inclusions, classifications, and multiple and hierarchical classifications).
Achievement of a score in this test requires three to twelve consistent decisions, depending on the type of task.

The Combinative Operations Test (Csapó 1988) involves twelve tasks based on combinatorial structures (variations with repetitions, variations without repetitions, variations with different numbers of elements, combinations, all subsets of a set, Cartesian product of two sets). Performance on the tasks is quantified using a formula that takes into account both the number of the properly constructed combinations (variations, etc.) and the number of errors. Attainment of the score requires two to five consistent decisions; the maximum test score is 170.

In the Logical Operations Test the underlying structures were provided by ten binary operations of propositional logic, where the truth-value of the complex proposition is a function of both propositions. By using these operations, two equivalent tests were devised. Both tests begin with a short story that introduces a situation (e.g., children sitting around a table and waiting for breakfast), and the ten tasks then examine the subject’s decisions about the truth-value of the complex propositions (which are statements in the given situation) containing a particular logical operation. Attaining a score requires four consistent decisions. Though the two versions are considered to be equivalent, to increase the reliability of the measurement both versions were administered to every child at both measurement points in two different testing sessions. In this way they can be used as mutual controls, and, where one of them was missing, its result was replaced by the other one. Their means were very close to each other: e.g., 35.28 for version A (n=923) and 34.19 for version B (n=919) for the pre-measurement of the fourth graders. During the course of data analysis, the twenty tasks in the two parallel tests are treated as tasks in a single test and their scores are summed (see Csapó 1987; Vidákovich and Csapó 1988).

The Raven’s Matrices test was also administered at both measurement points. All other tests and the data sheet were administered at the beginning of the school year before the beginning of the interventions. The test data presented in this paper are always percentages of the maximum test score.

Training materials

For each combination of the operational abilities to be developed, a collection of structured tasks was devised in co-operation with practising teachers. Specifically, this team-work resulted in collections of exercises that were edited and mimeographed in booklet form. Each of the twelve (grammar-combinative, grammar-logical...physics-systematizing) collections contained about fifty-three of these structured tasks. The fourth grade tasks for the systematizing ability were exceptions. These tasks were devised by using the analogy and experience of the other work and were not tried out and refined before their use in the main experiment. A
detailed description of the tasks and some examples can be found in Csapó (1987).

Interventions

The phenomena of ‘the zone of proximal development’ may exist in any educational system: there are some possibilities for improvement, but there are also some barriers that cannot be stepped over. In designing an experiment to try out methods that might improve the existing practice, we have to take these barriers into consideration. We have to consider not only what can be changed in the course of the experiment, but what can be changed in teaching practice as well.

In this experiment, the limited vocabulary of a possible metalanguage and the teachers’ modest knowledge of psychology reduce the range of application of the metacognitive effects. However, the fact that during their study academic training outweighs the psychological and educational preparation offers only one, albeit stable way, to communicate everything the teachers are expected to do in the terms and categories of the specific subject they teach. The teachers of the intervention classes had not received any special training. After being provided with the intervention material, they were given a brief explanation of the aims of the research and the activity they were expected to do.

During the interventions, the use of mathematical concepts or terms was avoided. This method does not wish to assume part of the role of mathematics teaching or to be disguised maths training. No direct rules were taught (in contrast with many intervention studies, where a specific performance or skill was probably learnt, e.g. in the study by Fishbein et al. 1970). The training was exclusively limited to combining, classifying, etc., the concepts of teaching material in a meaningful way in the particular context or discussing why the complex propositions of the texts are true or false.

The structured tasks could be applied in several ways: in individual, in group, or in whole-class work. During the pilot studies teachers retained the right to choose their methods of working, the only requirement being that each child in the experimental classes should deal with the given fifty exercises during the school year. Decisions concerning the form and materials of the interventions of the main study were made in co-operation with teachers on the basis of reports of the pilot studies. While teaching stereotypes limited the possibilities of using the structured tasks, usually one experimental lesson per week and two tasks per lesson (one at the beginning of the lesson as whole-class work and one at the end of the lesson as individual work) should be considered as a rule. As extra time could not be devoted to the interventions, the structured experimental tasks were alternative versions rather than additions to school subjects. The school year begins on 1 September, and this month was chosen for the pre-
measurements. Thus, the interventions lasted practically eight months (October to May), and the post-tests were administered within the last two weeks.

Teachers submitted written reports after each experimental lesson. These documents were gathered and could also be analysed. They are important sources for the reshaping of the task-systems and the design of further applications.

RESULTS AND DISCUSSION

Before presenting the results of training, it is worth summarizing the students’ characteristics concerning operational thinking as measured in the ability tests at the beginning of the experiment. Table 8.2 presents the means and standard deviations for the two ages, involving all experimental and control groups. The ratios of the achievements in the fourth and seventh grades are also given to compare the pace of development within this age range.

The systematizing ability shows the greatest change between the ages of 10 and 13, and the development of logical ability is very slow. As the starting mean score for Raven’s Matrices is over 75 per cent for the seventh graders, the ceiling effect may influence the test results and part of the change cannot be detected at this age. The effects of interventions can be evaluated by comparison of the groups.

While the change during the intervention period is a function of the starting level, for a reliable comparison particular attention must be paid to the equal initial levels of the samples compared. For example, in the fourth-grade control a correlation of \( r = -0.35 \) was found between the starting value and the change score. While children with a lower developmental level benefit more from the treatment, this relationship is even stronger in the experimental groups. As mentioned before, the required similarity of the groups could not be reached by selecting the classes. Thus the groups to

<table>
<thead>
<tr>
<th></th>
<th>Combinative</th>
<th>Logical</th>
<th>Systematizing</th>
<th>Raven</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>4th grade (n&gt;900)</td>
<td>51.6</td>
<td>21.2</td>
<td>39.4</td>
<td>14.7</td>
</tr>
<tr>
<td>7th grade (n&gt;950)</td>
<td>63.6</td>
<td>19.7</td>
<td>45.1</td>
<td>14.3</td>
</tr>
<tr>
<td>7th/4th grade ratio</td>
<td>1.23</td>
<td>1.14</td>
<td>1.42</td>
<td>1.21</td>
</tr>
</tbody>
</table>
be compared were matched in the course of the data analysis. The computer program compared the frequency distributions of the two groups and chose equal numbers of subjects from each achievement interval for inclusion in the matched groups. The data presented here are results of comparisons based on 10 per cent intervals. In this way, the difference between the pre-test means is always less than 1 per cent, and also the shapes of the frequency distributions of the compared groups were very similar.

**The effects of training on the abilities in question**

During the data procedure, a change score was defined as the difference between the results of post- and pre-tests. A series of t-tests was carried out in order to test the significance of the difference between the means of change scores. First, we summarize the overall results of the experimental groups in a way that allows us to compare the effects of the various trainings with each other and with results published elsewhere. The effect size that is often used in meta-analysis literature is the most appropriate for this purpose (see Chapters 6 and 9).

Table 8.3 presents the size and significance of the effect of training on the students’ targeted operational abilities. These data can be compared with the near-near effect sizes reported by Goossens (Chapter 9, this volume). In the fifty-nine training studies he examined, Goossens found an
average effect size equal to .74. The effect size for the nineteen long-term interventions was equal to .51.

Although there are large differences in the sizes of the various effects, our results are systematic in the sense that they demonstrate significant effects in almost every combinative experimental group, and only at the fourth grade in the logic experimental groups. No significant positive effect was found in the systematizing training groups. These results suggest that the three operational abilities are not equally sensitive to training. It is also evident that there is an interaction between age and abilities.

In order to have a look into the background of these characteristics, we have to place our data in other contexts. Shayer (Chapter 6, this volume) suggests that we should compare the changes caused by the intervention with a baseline of normative data which describes the development of the child population. Although we do not have as sophisticated a body of data to draw a baseline as in the medical studies cited by Shayer, we do have measurements for two ages with relatively large samples (Table 8.2). These measurements allow us to draw a draft estimation of development in the range examined here. Figure 8.1 presents the results of training in this context (only for groups with one kind of training).

The developmental lines of combinative experimental groups and matched control groups are displayed in Figure 8.1a together with the baseline estimated from the cross-sectional measurements. The developmental lines of the matched control groups are almost parallel with the baseline. The development is faster and the effects are larger in those groups where the developmental level was lower at the beginning of the experiment. The differences in change are significant in all three fourth-grade groups. In both experimental groups, the size of the effect is greater when the training took place only in one school subject (Table 8.3). In the seventh grade, the only group not to show a significant gain was the one in which the children were trained in two subjects. When the gains of the
experimen
tal groups at the two ages are compared, the results consistently show that the training applied in combinative reasoning is more effective at the younger age.

At both ages it was found that training with fifty exercises during eight months has a greater (measured) effect than the use of a hundred tasks. While it is not probable that an ability that is improved by a certain kind of training will be destroyed by further training of the same kind, we presume that too many exercises in the same type of activity becomes boring, attention is reduced, and this negative attitude influences the results of the testing too. An exercise with combinative structures required about twice the time needed for an exercise concerning propositions. Thus, the smaller measured gain here is understandable.

Figure 8.1b shows the results of the propositional training on the 10- and 13-year-old children’s logical ability. In every fourth-grade experimental group significant gains were found, and almost zero effect sizes at seventh grade. As the figure indicates, at seventh grade the development of all groups falls into the same line, which is identical with the baseline. It is not possible to re-open here the discussion about the appropriateness of the framework of propositional logic in investigations of cognitive development. Some leading theorists argue against it (e.g., Johnson-Laird 1983), while many others use it (e.g., Seggie 1978). Our view is close to the conclusion drawn by Lawson after his review of a large amount of research into the development of formal reasoning: ‘Propositional logic is not isomorphic with advanced reasoning, thus without modification it serves as an unsatisfactory model of the thought processes of the advanced formal thinker’ (Lawson 1985:609).

The results of the experimental groups related to systematizing ability are displayed in Figure 8.1c. No significant gains for the experimental groups were found in these cases (except for a negative one, which is probably due to testing errors). The developmental lines of the matched control groups run parallel with each other and, unlike the other abilities, indicate greater changes in these groups than would be expected with regard to the cross-sectional estimation. This may indicate that the testing procedure itself improves to some degree the children’s performance. The operations classified in this group appear in the pre-formal stages in the Piagetian model. However, earlier measurements (Nagy 1988) indicate that a considerable proportion of children did not master these structures by the age of 14. Therefore, it seemed promising to try to accelerate their development.

As mentioned before, the systematizing operations develop quickly in the age range examined. The changes displayed in Figure 8.1c suggest that large changes can be found in all groups from pre- to post-tests. When the development of the three operational abilities in the control groups is taken into account, the changes in the systematizing ability are much greater than those observed in the other two abilities. Two different factors
may contribute to these findings. On the one hand, systematizing operations are very sensitive to training. On the other hand, the standard way of teaching covers a wide range of activities that stimulate the development of the systematizing operations.

When interpreting the results of the interventions, we have to take into consideration the argumentation of Shayer and Beasley (1987). In their study on instrumental enrichment, they concluded that the results of the interventions should be evaluated rather on the fresh learning embarked on after the intervention had ended. We share this view, but it must be mentioned that the delayed effect is greater if the training is aimed at affecting the learning potential. In the present experiment, long-term effects are also considered to be more important than the effects measured at the end of the interventions. Therefore, further investigations are planned. The aim is to study how the learning potential can be influenced and what the effects of the training are over a longer period of time. The range of effects can in part be estimated by studying their transfer on to other variables.

The effects of training on other abilities

As we administered the same set of tests to all groups, it is possible to study how the training affected the other operational abilities. These effect sizes can be compared with the 0.38 mean near-far effect size reported by Goossens (Chapter 9) for long-term interventions.

The greatest transfers were found in the logic experimental group. In the fourth-grade grammar ($\sigma=0.57, p<0.01$) and grammar+science ($\sigma=0.39, p<0.05$) groups, and in all the three seventh-grade groups (chemistry: $\sigma=0.58, p<0.01$; physics: $\sigma=0.70, p<0.001$; chemistry+physics: $\sigma=0.59, p<0.001$), significant gains were found in the combinative tests. The systematizing ability was significantly affected by the propositional training in only one group (seventh-grade chemistry+physics: $\sigma=0.32, p<0.05$). The combinative training significantly affected the systematizing ability only in the fourth-grade grammar intervention group ($\sigma=0.45, p<0.01$).

The training devised to accelerate the development of systematizing operations significantly affected the development of combinative operations in the seventh-grade physics group ($\sigma=0.36, p<0.05$). There was also one seventh-grade group (chemistry+physics: $\sigma=0.33, p<0.05$) where logical ability was significantly influenced by the training in systematizing abilities.

These findings do not offer a consistent view of the transfer effects. However, the transfers found clarify the picture: they show that some training procedures affected other operational structures more significantly than the targeted ones. As a significant effect was found in two of the six groups trained with systematizing material, we can drop the hypothesis that these materials are devoid of any effect. It was also shown that the seventh-grade logic training material did improve the thinking, even if its
effect was not detected in the logic test; this effect was indicated by the combinative tests in all three groups.

The effects of training on intelligence

Significantly greater changes in the intelligence test were found in some of the fourth-grade intervention groups than in the control groups. Two-thirds of the combinative and logical training groups showed meaningful gains (the mean $\sigma=0.27$ for these six groups), while none of the systematizing training material influenced significantly the achievements in the intelligence test.

In the seventh-grade experimental groups, no significant gains were found compared to the controls in the Raven test. At the age of 13, the starting achievements in Raven’s Matrices were over 75 per cent, so at this age the test is not sensitive enough to detect the changes in fluid intelligence. Thus, on the basis of these results it cannot be determined whether intelligence was affected by the training at the age of 13.

Improving intelligence significantly cannot be a realistic aim for an intervention within eight months. However, the changes detected in the fourth grade suggest that the enrichment of the learning material with combinative and logical structures around the age of 10, and the use of this material over a period of years, might accelerate the development of intelligence.

GENERAL DISCUSSION

From a comparison of the three groups of operations, it was found that the systematizing ability shows the fastest development in the age range studied. The development of this ability was not accelerated by the means applied in the experiment. The structured task systems devised to improve these abilities had weak effects on the other abilities. Of course, it might be argued that the testing method of treatment applied was not the proper one, or that an effect could be detected only after a considerable period of time. However, we prefer to adopt the view that this group of operations receives the strongest developmental influences from the standard teaching context. Therefore, there is no room for their acceleration by the methods tested by the present study.

Logical operations develop slowly, and they can be improved at a young age. The intervention applied has a significant effect on the other abilities at the older age, too. Thus, the intervention did improve thinking, but this change was not detected by the measurement applied. This fact supports the view that the improvement in advanced formal thinking cannot be characterized in terms of formal logic.

On their own, combinative operations develop at an intermediate pace. Because of the application of our structured exercises, a significant
acceleration can be achieved at both ages. The standard teaching material does not contain enough stimulation for their development, and while children do not reach the end-stage by the age of 13, an enrichment of the teaching material with these structures seems to be worthwhile.

While the effects of the comparable interventions were very different on the examined three abilities, the training study indicates that the differences between these abilities are more than their similarities. The lower near—far transfers also support the view of domain-specific operational structures rather than the concept of universal thinking structures.

In most of the fourth-grade experimental groups (fluid) intelligence measured on Raven’s Matrices was significantly improved by the interventions in combinative and logical operations. Although a direct improvement in intelligence cannot be the aim of the exercises in some specific structures, better operational thinking facilitates subsequent learning. Thus, rather a long-term effect on intelligence can be expected.

From the point of view of the training material, an enrichment with combinative and logical structures seems to be effective. The systematizing structures have much weaker effects, though in some cases they are detectable. This poor influence is not only a question of quantity (twice the number of interventions did not have a greater effect); it must have structural causes.

Although the present mathematics curriculum includes the rudiments of set theory, mathematical logics, and combinatorics, and such exercise can be found in mathematics work-books, experiences with these structures in a wide range of the context areas does facilitate the development of operational thinking. The new mathematics teaching is faced with the problem of finding real contents from everyday life that children are familiar with to exercise its specific structures. On the other hand, other school subjects, primarily in the first grades, search for ways to exercise their specific concepts, propositions, facts, and relationships. Enriching the teaching material with structured tasks might be one step towards bridging the gap between abstract mathematical structures and everyday activity with concrete contents in other school subjects. In this way, a better share of the work could be reached between mathematics and other subjects in the development of operational thinking.

The acceleration attainable within a school year is limited. After an acceptable number of the exercises, they no longer result in further development. Too much activity with the same kind of structure may even have a negative influence. In regular school practice, ten to twenty exercises in the same group of operations might be enough per school year, and their total number must not exceed thirty. Consequent enrichment of the teaching material with this activity in several school subjects over subsequent years may rather be the best way to apply these exercises in educational practice.
The results suggest that students with less mature operational thought benefit more from the training chosen than do the more mature children. Training might be more economic if these less mature students were targeted, but this requires more complicated classroom procedures.

ACKNOWLEDGEMENTS

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